Modelling Physicians' Recommendations for Optimal Medical Care by Random Effects Stereotype Regression

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Abstract: We show how Anderson's Stereotype regression model can be extended to account for correlated responses by a simple nonlinear parameter restriction on the multinomial logistic model with random effects. A data set on physicians' recommendations and preferences in traumatic brain injury rehabilitation is used for illustration.

Keywords: Stereotype regression; Random effects; Multinomial logistic regression; Traumatic brain injury

1 Introduction

Many study designs in applied sciences give rise to correlated data. For example, subjects are followed over time, are repeatedly treated under different experimental conditions, or are observed in logical units (e.g. clinics, families, litters).
One of the standard analysing tools in these situations which adequately accounts for the correlation between observations is the random effects (RE) model, sometimes also called hierarchical or mixed effects model. This is quite common for continuous responses, and also, despite an enhanced mathematical complexity, for binary responses. Less used have been random effect models for the analysis of discrete non-binary responses, some of the rare examples are Hedeker and Gibbons (1994) and Tutz and Hennevogl (1996) for ordinal and Hartzel et al. (2001) for nominal responses. To our knowledge, up to now there exists no random effects version of the Stereotype regression model.
The Stereotype regression model was originally proposed by Anderson (1984). He observed that some relevant discrete non-binary responses in applied statistics are not perfectly ordinal in the sense that there is an latent continuous variable which was only observed in discrete and disjunct classes, but should rather be regarded as a multidimensional phenomenon where several items determine the grade on the ordinal scale, the most prominent example being maybe the severity of a disease.

2 The Random Effects Stereotype Regression Model

To extend the Stereotype regression model to account for correlated responses we use the fact that the original Stereotype model is derived from the ordinary multinomial logistic regression model by a certain nonlinear parameter restriction. This restriction is simply applied to the multinomial logistic random effects model of Hartzel et al. (2001). The Stereotype model with random effects thus becomes a nonlinear model with random effects and all the well-known theory and estimation methods (see e.g. Davidian and Giltinan, 1995) can be used.
We assume that our data comprises a set of I $(i = 1, \ldots, I)$ independent clusters where the i-th cluster consists of n_i observations. Let Y_{ij} denote the j-th response in cluster i $(j = 1, \ldots, n_i)$, where this response is from one of r $(r = 1, \ldots, R)$ distinct categories and the response probability is $\pi_{ijr} = P(Y_{ij} = r)$. Further, x_{ij} denotes a column vector of covariates for the j-th observation in the i-th cluster. Thus the model equation is

$$\log\left(\frac{\pi_{ijr}}{\pi_{ijR}}\right) = \theta_r + x'_{ij}\phi_r\beta + u_{ir}, \quad r = 1, \ldots, R-1, \tag{1}$$

where the θ_r are constant terms, the scalars ϕ_r introduce a metric for the common effect of the covariates, where this effect is assumed constant across response categories. The influences of covariates are assessed through the components of $\beta = (\beta_1, \ldots, \beta_p)'$. The θ_r, the ϕ_r, and the β are considered to be fixed effects. For the random effects u_{ir} we assume a multivariate normal distribution with unstructured covariance matrix Σ, that is for $u_i = (u_{i1}, \ldots, u_{i,R-1})'$ we have $u_i \sim N(0, \Sigma)$.
For reasons of identification of parameters we restrict $\theta_R = 0$, $\beta_R = 0$, $u_R = 0$, $\phi_1 = 0$, and $\phi_R = 1$, so that interpretation of parameters is, analogous to the multinomial logistic model, with reference to the R-th category. Note that the model equation of the RE Stereotype regression model is derived from the multinomial logistic random effects model of Hartzel et al. (2001) by the non-linear parameter restriction $\beta_r = \phi_r\beta$.
The estimation of parameters is complicated by the fact that the likelihood function consists of a product of I integrals which can not be solved in closed form. Thus, numerical or stochastic integration are viable alternatives. Hartzel et al. (2001) suggest adaptive Gaussian quadrature as the preferred method for parameter estimation in this model class. As such, the model can be fitted conveniently with, for example, SAS PROC NLMIXED.

3 The Motivating Example

The motivation for the derivation of the RE Stereotype model was a data set from a study on physicians' recommendations and preferences in traumatic brain injury (TBI) rehabilitation (Hasenbein et al., 2003). In this study, 36 physicians were asked to decide on the optimal rehabilitation setting (in-patient, day-clinic, out-patient) for each of ten typical TBI disease histories. Of course, we expect the setting recommendations within the same physician to be correlated. Concerning the 3-valued response we recognize that this is not strictly nominal, but has indeed some ordinal flavor, for example, we might think of the "time not at home" as some underlying continuous variable. However, it is not that simple that in-patient, day-clinic, and out-patient rehabilitation only differ by the time that patients stay in the clinic, instead they rather represent different therapeutic concepts and actual treatment varies. Of interest was mainly if we could identify factors (considering physicians and disease histories) that influence setting preferences.

In the following (see Table 1) we give the results (estimates and respective standard errors in parentheses) for our data set for the ordinary Stereotype model, the RE multinomial model and the RE Stereotype model. Four covariates, all of them binary, were included in the model, two of them referring to physicians' characteristics (1. Is the physician a neurologist [NEURO] and 2. Is the physician a specialist [SPECIAL]) and two describing the disease history (3. Is the time since the event longer than 3 months [TIME] and 4. Is the patient severely or moderately handicapped after the TBI [SEVERITY]). As the reference category of the response we chose the stationary setting, and compare day-clinic (DC) and out-patient (OP) to this.

Some remarks regarding the results can be made: As we expect (and maybe hope as potential patients), physicians' own characteristics do have only small influence on their recommendations. Looking at the values of the model selection criteria we see that the random effects Stereotype model is superior to the other two models: Compared to the ordinary Stereotype model this means on one hand that it is essential to account for the inherent correlation in the data (which is also confirmed by the significant values of the random effects covariance matrix). Compared to the random effects multinomial model on the other hand we note that we do not need the additional information of looking separately at the two response categories, instead the RE Stereotype model gives a natural summary of the ordering of response categories and judges the DC category roughly in the middle ($\phi_2 = 0.55$) between the reference category and the OP category. Summing up a bit roughly in subject matters: The more severe the TBI and the shorter the time since TBI, the more time the patient should spend in the hospital.

4 Discussion

We showed how Anderson's Stereotype regression model can be extended easily to account for correlated responses. The idea was to impose the non-linear parameter restriction which relates the ordinary multinomial logistic

TABLE 1. Results (estimates and respective standard errors in parentheses) from the ordinary Stereotype model, the RE multinomial model and the RE Stereotype model for the TBI data set

	Stereotype Model	RE Multinomial Model		RE Stereotype Model
		Fixed effects		
		DC	OP	
$\hat{\beta}_{NEURO}$	0.89 (0.46)	-0.56 (0.74)	-0.36(0.90)	1.36 (0.93)
$\hat{\beta}_{SPECIAL}$	0.19 (0.43)	-0.63 (0.78)	-0.04 (0.94)	0.40 (0.88)
$\hat{\beta}_{TIME}$	3.26 (0.45)	2.51 (0.41)	3.43 (0.50)	4.23 (0.58)
$\hat{\beta}_{SEVERITY}$	-2.00 (0.43)	-1.94 (0.43)	-3.29 (0.47)	-2.60 (0.53)
$\hat{\phi}_2$	0.50 (0.10)	–	–	0.55 (0.09)
		Random effects		
$\hat{\sigma}_1^2$	–	1.47 (0.35)	–	2.01 (0.96)
$\hat{\sigma}_2^2$	–	–	1.87 (0.43)	2.62 (1.20)
$\hat{\sigma}_{12}^2$	–	2.54 (1.11)		1.94 (0.93)
		Model selection criteria		
AIC	517.3	499.3		487.5
BIC	543.9	516.7		503.3

model to the original Stereotype model to the random effects multinomial model of Hartzel et al. (2001). Proceeding that way, the RE Stereotype model becomes a nonlinear random effects model and standard theory and estimation methods apply. In terms of our motivating example we were able to identify factors which influence physicians' preferences on optimal rehabilitation setting in TBI patients. We learned that we had to account for the inherent correlation in the data but did not need the additional complexity of the RE multinomial model. Moreover, we got information about distances between response categories. The estimation method of numerical integration seems to work well as some limited preliminary evidence from simulation studies reveals. In the future we are mainly interested in additional estimation techniques to judge robustness of our results, where MCMC and nonparametric ML methods might be promising candidates.

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